

From Independent to Coupled Hysterons: The Preisach Principle

📅 Sep 24, 2026 · @Danny

Abstract

We extend the Preisach model in three steps.

1. **The classical model (Preisach, 1935).** The output is a weighted superposition of independent bistable relays, with a density $\mu(\alpha, \beta)$ that is fixed.
2. **Van Hecke's extension.** Following van Hecke and co-workers, we allow each relay's thresholds to depend on the states of the other relays through a coupling matrix. That matrix may be signed and asymmetric.
3. **Our extension, the Preisach principle.** We propose that the density itself should be treated as coupling-shaped: a functional of the system's configuration and a coupling kernel, rather than a material constant.

This formulation contains the classical model, mean-field feedback models and discrete interacting-hysteron models as special cases. We further propose it as a domain-general modeling hypothesis. Under that hypothesis, any population of agents or propositions that switch between two committed states under a common drive, with mutual influence on each other's switching thresholds, can be written in the same threshold-and-coupling form.

We illustrate the formalism with a six-hysteron system: two internally coupled triads joined by asymmetric cross-couplings. We state explicitly which parts are established physics and which are proposal.

Scope. This document covers the formalism and one worked example only. Illustrative case studies are kept in a separate illustration set.

1. The classical Preisach model

The classical model describes a hysteretic output $f(t)$ as a weighted superposition of independent relays driven by a common scalar input $u(t)$.

1.1 The hysteron

A hysteron is a relay operator $\gamma_{\alpha\beta}$ with an up threshold α and a down threshold β , where $\alpha \geq \beta$. It takes values in $\{-1, +1\}$:

$$\hat{\gamma}_{\alpha\beta}[u](t) = \begin{cases} +1 & u(t) \geq \alpha \\ -1 & u(t) \leq \beta \\ \hat{\gamma}_{\alpha\beta}[u](t^-) & \beta < u(t) < \alpha \end{cases}$$

Inside the window $\beta < u < \alpha$ the relay is bistable, and its state is set by the last threshold it crossed. That dependence on history is the only memory in the model.

1.2 The Preisach density and the Preisach triangle

The output is a weighted integral over a population of relays:

$$f(t) = \iint_T \mu(\alpha, \beta) \hat{\gamma}_{\alpha\beta}[u](t) d\alpha d\beta, \quad T = \{(\alpha, \beta) : \alpha_0 \geq \alpha \geq \beta \geq \beta_0\}$$

- **The Preisach density.** $\mu(\alpha, \beta) \geq 0$ is the Preisach density. It is a fixed property of the material, independent of u and of the system's state.
- **The Preisach triangle.** T is the Preisach triangle in the (α, β) half-plane. Its hypotenuse is $\alpha = \beta$, and its corners are set by the saturation fields α_0 and β_0 .

At any time, T splits into a region $S^+(t)$, where relays are up, and a region $S^-(t)$, where relays are down. Therefore

$$f(t) = \iint_{S^+(t)} \mu d\alpha d\beta - \iint_{S^-(t)} \mu d\alpha d\beta$$

The boundary $L(t)$ between the two regions is a staircase. Increasing u sweeps a horizontal line up in α , and decreasing u sweeps a vertical line left in β . The vertices of the staircase are exactly the input extrema that have not been wiped out, so $L(t)$ is the model's entire memory.

1.3 Two structural properties

- **Wiping out.** Each new input maximum erases the vertices of all smaller previous maxima, and each new minimum erases all larger previous minima. The model stores alternating dominant extrema, not the full input history.
- **Congruency.** All minor loops traced between the same two input values u_1 and u_2 are congruent, whatever history preceded them.

Mayergoyz showed that these two properties are necessary and sufficient for a hysteresis operator to be represented by the classical Preisach model. They are also

where the model is known to fail for some real materials, whose minor loops are not congruent.

1.4 The assumption we relax

The relays do not interact. μ depends only on (α, β) , and each relay's thresholds are fixed constants that do not depend on the states of the other relays. This independence makes the model linear in μ and identifiable from first-order reversal curves (the Everett function). It is also the assumption the rest of this document removes.

2. Interacting hysterons (van Hecke)

Van Hecke's extension keeps the relay but makes its thresholds depend on the configuration of the other relays. With only three hysterons, this produces more than 15,000 distinct transition graphs, and most of them violate return-point memory ([van Hecke, Phys. Rev. E 104, 054608, 2021](#)).

2.1 Coupled thresholds

Take N hysterons with states $s_i \in \{0, 1\}$, which relate to the classical γ by $s = (1 + \gamma)/2$. Collect them in the configuration $S = (s_1, \dots, s_N)$. Each hysteron has bare switching fields $u_i^+ > u_i^-$. Coupling makes the effective fields depend on the state:

$$u_i^\pm(S) = u_i^\pm + \sum_{j \neq i} c_{ij} s_j$$

The coupling coefficients c_{ij} carry no symmetry or sign constraints:

- c_{ij} may be positive or negative.
- In general $c_{ij} \neq c_{ji}$.
- Diagonal terms are absent.

In our sign convention, $c_{ij} < 0$ means that hysteron j being up makes i easier to switch up and harder to switch down (cooperative). $c_{ij} > 0$ is antagonistic. Sign conventions differ between papers, and only the physics matters here.

One consequence of this form: both fields of hysteron i shift by the same amount, so the width of its bistable window, $u_i^+ - u_i^-$, does not depend on S . Coupling moves the window. It does not squeeze it.

2.2 Stability, avalanches and transition graphs

A configuration S is stable at drive U if every hysteron is consistent with its own effective

window:

$$s_i = 1 \Rightarrow U > u_i^-(S), \quad s_i = 0 \Rightarrow U < u_i^+(S) \quad \forall i$$

For each S , this defines a stability interval $(U^-(S), U^+(S))$, which may be empty. Under quasistatic driving, the system stays in S until U leaves that interval. One hysteron then flips. The flip changes every other hysteron's thresholds, and this can destabilise further hysterons at the same U . The resulting cascade is an **avalanche**.

When several hysterons are unstable at once, the model needs an update rule; a common choice is to flip the most unstable one first. The model's predictions can depend on this choice, so any quantitative claim should state the rule used.

The **transition graph** (t-graph) has the stable configurations as nodes, with one up-edge and one down-edge leaving each node. It is the complete description of the system's response to arbitrary quasistatic driving.

2.3 What coupling changes

Property	Independent (Preisach)	Interacting (van Hecke)
Switching order	Fixed by the ordering of bare fields	State-dependent (scrambling)
Flips per threshold crossing	One	One or many (avalanches)
Return-point memory	Always holds	Violated in most t-graphs
Distinct t-graphs, $N = 3$	A small, closed class	More than 15,000

Known exception: purely cooperative coupling. Return-point memory is not lost for every choice of coupling. When all couplings are cooperative ($c_{ij} \leq 0$ in our convention), the dynamics obey Middleton's no-passing property, which guarantees return-point memory. This is the mechanism behind return-point memory in the zero-temperature random-field Ising model (Sethna et al., 1993). Violations therefore require at least one antagonistic (frustrated) coupling, or a non-monotone update rule. This is directly relevant to the two-triad example below, which is built with both signs on purpose.

3. The Preisach principle

The Preisach principle has two parts: a formal claim and a modeling hypothesis.

- **Formal claim.** The Preisach density should be read as coupling-shaped. It is a functional of the system's configuration and a coupling kernel, not a fixed material function.

- **Modeling hypothesis.** This threshold-and-coupling structure is proposed as a common language for interacting parties outside physics.

The name credits Preisach's 1935 model and is kept deliberately distinct from it. The model is Preisach's. The principle is a guideline for applying its structure where coupling is present.

3.1 Coupling-shaped density

Start from a bare population density $\mu_0(a)$ on the Preisach triangle, where $a = (\alpha, \beta)$. Let $s(a, t) \in \{0, 1\}$ be the state of the hysteron born at a . Introduce coupling kernels $K^\pm(a, a')$. These give the threshold shift that the hysteron at a' , when up, induces on the hysteron at a :

$$\Phi^\pm[s](a) = \iint_T K^\pm(a, a') s(a') \mu_0(a') da'$$

Each hysteron's effective thresholds become $\alpha + \Phi^+s$ and $\beta + \Phi^-s$. The population density the drive actually sees is the pushforward of μ_0 under this state-dependent displacement:

$$\mu_s(\alpha, \beta) = \iint_T \mu_0(a') \delta(\alpha - \alpha' - \Phi^+[s](a')) \delta(\beta - \beta' - \Phi^-[s](a')) da'$$

The output keeps its classical form, with the fixed μ replaced by the state-dependent $\mu_s(t)$:

$$f(t) = \iint \mu_{s(t)}(\alpha, \beta) \hat{\gamma}_{\alpha\beta}[u](t) d\alpha d\beta$$

The state s is itself determined by μ_s , so the problem is self-consistent. At fixed u , a stable configuration is a fixed point of the switching rule under its own displaced thresholds. An avalanche is a discontinuous jump between such fixed points as u varies.

3.2 Special cases

The classical model, mean-field feedback and van Hecke's discrete model are all special cases of this one structure. They differ only in the kernel and in the support of μ_0 :

Kernel $K^\pm$	Bare density μ_0	Recovers
δ	Continuous	Classical Preisach model ($\mu_s = \mu$)

9/25/26, 4:39 PM

From Independent to Coupled Hystérons: The Preisach Principle

	Continuous	Classical Preisach model ($\mu_0 = \mu_0 / 2$)
Constant k , the same for all a	Continuous	Mean-field feedback, i.e. a rigid shift of the triangle with net magnetisation (Della Torre's moving Preisach model)
General	Sum of N delta functions	Van Hecke's interacting hystérons, with $c_{ij} = K(a_i, a_j)$
General	Continuous	The coupling-shaped case proposed here

The last row is what is new. The density acquires state-dependent shape, not only a state-dependent offset. A cluster of hystérons can be pulled across the diagonal of the triangle, or compressed toward it, by the configuration of another cluster.

3.3 Admissibility conditions

Three conditions keep the construction well defined. A physicist should check them for any proposed instance:

- Bistability is preserved.** Each hysteron needs $\alpha + \Phi^+ \geq \beta + \Phi^-$ in every reachable configuration. With $K^+ = K^-$, as in Section 2, this holds automatically. With $K^+ \neq K^-$, a hysteron can be pushed across the diagonal, where it has no stable state inside its window and can oscillate.
- The update rule is stated.** Avalanche outcomes, and therefore the t-graph, can depend on the order in which simultaneously unstable hystérons flip.
- Driving is quasistatic.** The formalism assumes that switching is fast compared with the drive. Rate-dependent effects are outside it.

3.4 The domain-general hypothesis

The hypothesis is that interactions between parties can be modeled by assigning the following four elements. The parties may be aware or unaware, and may be people, ideas, AI systems or groups.

- **A binary committed state for each party.** For example, has or has not adopted a position, or has or has not acted.
- **A scalar drive U common to them.** This may be evidence, pressure, load or confidence.
- **A bistable window for each party.** The window is (u_i^-, u_i^+) .
- **A coupling matrix.** It records how one party's commitment moves another's thresholds.

A particular choice of these elements is an *instance* of the principle. The principle fixes the structure, and each instance supplies the parameters. The Linux analogy in our

working notes compares the principle to a kernel and each instance to a distribution.

The hypothesis has prior art. Threshold models of collective behaviour (Granovetter, 1978) and random-field Ising models of opinion shifts are close relatives. What we add is the explicit hysteron and t-graph framing. With that framing, the diagnostic signatures from physics carry over directly: avalanches, scrambled switching order and return-point memory violation.

The hypothesis is not a result. It commits to binary states, a low-dimensional drive, deterministic thresholds and quasistatic dynamics. Each of these can fail in a given domain, and each such failure is a way to falsify an instance.

4. Worked example: the two-triad system

Six coupled hysterons in two internally coupled triads show avalanches, a scrambled release order, and a mixed state that the uncoupled system never reaches. Return-point memory survives every history we tested. This section fixes the parameters, reports the transition graph, and compares it with the same six hysterons at zero coupling.

4.1 Set-up

The system has two triads:

- **Negative cluster D.** Hysterons 1–3 switch up under increasing drive.
- **Positive cluster K.** Hysterons 4–6 switch up under decreasing drive.

To handle the opposite response, we give each hysteron a polarity σ_i and replace U by the local field $\sigma_i U$ in the stability condition of Section 2.2. D has $\sigma = +1$ and K has $\sigma = -1$. The drive U can be read as the gap between asserted confidence and verified competence, but nothing below depends on that reading.

$$s_i = 1 \Rightarrow \sigma_i U > u_i^-(S), \quad s_i = 0 \Rightarrow \sigma_i U < u_i^+(S), \quad \sigma = (+1, +1, +1, -1,$$

The bare fields are in units of U . Within each triad they are staggered, so without coupling each triad switches one member at a time.

i	Cluster	Interpretive label	u_i^+	u_i^-	Up when
1	D	Dunning-Kruger	1.0	0.2	$U \geq 1.0$
2	D	Cobra	1.6	0.6	$U \geq 1.6$
3	D	Tacoma	2.2	1.0	$U \geq 2.2$
4	K	Schwarzenegger	0.4	-0.4	$U \leq -0.4$
5	K	Walt Whitman	1.0	0.2	$U \leq -1.0$
6	K	Walt Whitman	1.6	0.6	$U \leq -1.6$

D	K	vvv vvgt	1.0	0.2	U ≤ 1.0
6	K	Kaizen (no-ceiling ascent)	1.6	0.6	U ≤ -1.6

The labels are an interpretive layer. The analysis depends only on the numbers.

4.2 Coupling matrix

In the coupling matrix, row i is the hysteron being shifted and column j is the source. We use the convention of Section 2.1, where a negative entry is cooperative.

$$C = \left(\begin{array}{ccc|ccc} 0 & 0 & 0 & +0.8 & -0.5 & 0 \\ -0.8 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.8 & 0 & 0 & 0 & 0 \\ \hline +0.5 & 0 & -1.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.6 & 0 \end{array} \right)$$

The matrix encodes three kinds of coupling:

- **Intra-cluster chains.** These are feed-forward and cooperative. Within D, $1 \rightarrow 2 \rightarrow 3$ with $c = -0.8$. Within K, $4 \rightarrow 5 \rightarrow 6$ with $c = -0.6$. Each switch that goes up makes the next one easier to push over.
- **Cross-couplings.** These are asymmetric, and no entry equals its transpose:
 - $c_{43} = -1.6$: collapse (3) eases the declared-weakness stroke (4).
 - $c_{14} = +0.8$: follow-through (4) raises the bar for overconfidence (1).
 - $c_{41} = +0.5$: overconfidence (1) raises the bar for declaring weakness (4).
 - $c_{15} = -0.5$: frictionless momentum (5) lowers the bar for overconfidence (1).
- **Frustration.** The K cluster has the opposite polarity, so relabel it as $t_j = 1 - s_j$ to make every hysteron respond to +U. This flips the sign of every D–K coupling. In that frame, c_{15} and c_{43} are antagonistic. The directed loop $1 \rightarrow 4 \rightarrow 5 \rightarrow 1$ has one antagonistic link, so the system is frustrated. By the no-passing argument of Section 2.3, return-point memory is therefore not guaranteed in advance.

4.3 Transition graph

We drove the system quasistatically from negative saturation. When several hystérons were unstable at once, we flipped the most unstable one first. States are written $s_1s_2s_3 \mid s_4s_5s_6$. Of the 64 configurations, 17 have a non-empty stability interval, and 6 are reachable from saturation.

State	Stable for U in	Up-transition	Down-transition
-------	-----------------	---------------	-----------------

000 111	$(-\infty, 0.0)$	$U = 0.0 \rightarrow 000 110$ (flips 6)	none
000 110	$(-1.0, 0.4)$	$U = 0.4 \rightarrow 000 000$ (flips 5, 4)	$U = -1.0 \rightarrow 000 111$ (flips 6)
000 000	$(-0.4, 1.0)$	$U = 1.0 \rightarrow 110 000$ (flips 1, 2)	$U = -0.4 \rightarrow 000 110$ (flips 4, 5)
110 000	$(0.2, 1.4)$	$U = 1.4 \rightarrow 111 000$ (flips 3)	$U = 0.2 \rightarrow 000 000$ (flips 1, 2)
111 000	$(0.7, \infty)$	none	$U = 0.7 \rightarrow 011 100$ (flips 4, 1)
011 100	$(0.6, 1.8)$	$U = 1.8 \rightarrow 111 000$ (flips 1, 4)	$U = 0.6 \rightarrow 000 000$ (flips 2, 3, 4)



In the diagram, parentheses list the hysterons in the order they flip within the avalanche.

4.4 What the coupling does

To isolate the effect of coupling, we compared the same six hysterons with $C = 0$. That system reaches 7 states, the two independent ladders, and every transition flips exactly one hysteron.

- **Avalanches.** Seven of the ten coupled transitions flip two or three hysterons at a single field. Dunning-Kruger brings Cobra with it at $U = 1.0$, and Schwarzenegger brings Wu Wei at $U = -0.4$. The intra-cluster chains also move thresholds. Tacoma's up-field falls from its bare 2.2 to 1.4, because Cobra being up lowers it by 0.8.
- **A mixed state.** Lowering the drive from full D does not unwind D in stack order. At $U = 0.7$, the collapse coupling c_{43} pulls Schwarzenegger up first. Its antagonistic coupling c_{14} then knocks out Dunning-Kruger. The system lands in 011 | 100: Cobra and Tacoma are still latched and Schwarzenegger is on. No history can produce this configuration in the uncoupled model, where D and K states never coexist.
- **Scrambled order.** Uncoupled, D releases last-in-first-out: 3 at $U = 1.0$, then 2, then 1. Coupled, the first D hysteron to release is 1, and it goes via 4. Hysterons 2 and 3 follow only at $U = 0.6$, as one avalanche that also takes 4 down with them. The switching order depends on the state, as in Section 2.3.
- **Threshold for the cross-over.** With $c_{43} = -1.0$ instead of -1.6 , the mixed state disappears. Full D then releases directly to neutral at $U = 0.2$ in a single avalanche of 1, 2 and 3. The cross-cluster route opens somewhere between these two values.

2 and 3. The cross cluster route opens somewhere between these two values.

- **Return-point memory holds.** We tested 6,734 reversal histories: saturation, then two to three alternating extrema on a grid of step 0.5 over $[-3, 3]$, then an inner sub-loop. None violated return-point memory, for either C or $C = 0$.

The last point is itself a result. Frustration makes return-point memory violation possible, but it does not make violation inevitable. The six-hysteron system reproduces the avalanche and scrambling phenomenology of Section 2 without the memory loss. Finding the frustration strength at which return-point memory first fails is the natural next computation.

5. Open questions and limitations

The formalism in Section 3 is well posed, and the example in Section 4 is reproducible. The domain-general hypothesis is untested. The open problems below fall into three groups: mathematical, empirical and scope.

5.1 Mathematical

1. **Where return-point memory breaks.** Sweep the antagonistic couplings of Section 4.2 in strength and map the first violation. More generally, measure the fraction of return-point-memory-violating t-graphs as a function of a frustration measure suited to asymmetric C .
2. **Dependence on the update rule.** Repeat Section 4.3 with sequential, random and synchronous updates, and report which edges change. Any t-graph that depends on the rule should be flagged.
3. **Identifiability.** In the classical model, first-order reversal curves determine μ uniquely. In the coupling-shaped model, it is not clear whether single-drive data can separate μ_0 from K . We expect not in general, which would call for multi-protocol experiments or priors on K .
4. **Unequal kernels.** With $K^+ \neq K^-$, the width of the bistable window depends on the state, and admissibility condition 1 of Section 3.3 can fail. The dynamics beyond that point (oscillation, or an effective rate dependence) are not described here.
5. **Continuum avalanches.** For continuous μ_0 and a smooth K , the self-consistency of Section 3.1 may have jumps. How the size distribution of those avalanches depends on K is a natural bridge to random-field Ising results.

5.2 Empirical

1. **A null model.** For any dataset of threshold-crossing decisions, fit an independent Preisach model and a coupled one, then compare the three signatures of Section 2.3: avalanches, scrambled order and return-point memory violation. The claim of

interaction is only as strong as the margin over the independent null.

2. **Operational definitions.** Applying the model requires measurable definitions of the binary states and of the drive U . Without them, the parameters of Section 4 are illustrative, not fitted.
3. **Independent measurement of the coupling.** Coupling estimated from the same transitions it is used to explain is circular. Where possible, a coupling should be measured by perturbing one party and recording the threshold shift in another.

5.3 Scope and limitations

- **The example is hand-built.** The parameters in Section 4 were chosen to illustrate the mechanism, not fitted to data. The interpretive labels (Dunning-Kruger through Kaizen) carry no weight in the analysis.
- **The numerical checks are limited.** They cover one update rule, a reversal grid of step 0.5 and histories of up to three extrema. A finer grid or longer histories could expose violations these tests missed.
- **The assumptions are restrictive.** They are binary states, one scalar drive, deterministic thresholds and quasistatic driving. Real parties plausibly need several drives, noisy thresholds and finite switching times. Each extension is a separate model.
- **The generality claim is conditional.** The Preisach principle states that such systems can be written in this form. It does not claim that any particular system is described by it. That has to be shown case by case, against the null model of 5.2.

We present this as a working structure to be refined, not a closed result.

References

- F. Preisach, “Über die magnetische Nachwirkung,” *Z. Phys.* 94, 277 (1935).
- I. D. Mayergoyz, “Mathematical models of hysteresis,” *Phys. Rev. Lett.* 56, 1518 (1986).
- E. Della Torre, “Effect of interaction on the magnetization of single-domain particles,” *IEEE Transactions on Audio and Electroacoustics* 14, 86 (1966).
- M. Granovetter, “Threshold models of collective behavior,” *Am. J. Sociol.* 83, 1420 (1978).
- J. P. Sethna et al., “Hysteresis and hierarchies: dynamics of disorder-driven first-order phase transformations,” *Phys. Rev. Lett.* 70, 3347 (1993).
- A. A. Middleton, “Asymptotic uniqueness of the sliding state for charge-density waves,” *Phys. Rev. Lett.* 68, 670 (1992).
- M. van Hecke, “[Profusion of transition pathways for interacting hysterons](#),” *Phys. Rev. E* 104, 054608 (2021).

All six citations (Preisach, Mayergoyz, Della Torre, Granovetter, Sethna and Middleton) have now been verified against their sources, alongside van Hecke. Only the Della Torre journal abbreviation needed correcting.